# Parameters of maxon-EC-Motors Driven By Servo Amplifiers With Sinus-Commutation

## 1. Introduction

The maxon-EC-motors are three-phase permanent-magnet synchronous motors (PMSM) with sinusoidal air-gap flux-density distribution. In the applications, two types of servo amplifiers are mainly used to feed the currents in the three-phase windings.

- Servo amplifiers with block current waveform output, or block-commutation, feeding a dc current in two of three phase windings according to rotor position. This type of servo amplifiers is often used to feed a brushless dc motor having trapezoidal flux-density distribution and has advantages of simple control circuit and low cost. However, it causes torque ripple in the output magnetic torque of a three-phase PMSM.
- Servo amplifiers with sinusoidal current waveform output, or sinus-commutation, feeding three-phase sinusoidal currents in three-phase windings. Driven with this type of servo amplifiers, a PMSM can generate smooth output magnetic torque, and therefore, the control performance and as well as the acoustical noise can be improved the performance. The servo amplifier with sinus-commutation is suitable to realize high performance servo drives with maxon EC motors.

Up to now, the servo amplifiers with block commutation are widely used in the practical applications of electrical drives with maxon-EC-motors due to the low cost and the simplicity. The parameters such as torque constant, rating voltage and current, for the maxon-EC-motors in the data-sheets of maxon-EC-motor are oriented for these applications and measured with a servo amplifier with block-commutation. For the application of the servo amplifiers with sinus-commutation, these parameters will not be used directly to design an application system as well as to select the motor type and the voltage, current rating of servo amplifiers with the sinus-commutation.

With the development of electronics, micro-controllers and digital signal processors, the cost of a servo amplifier with sinus-commutation is getting so low that it can be comparable with a 4-Q servo amplifier with block-commutation. With the digital technology, the usage of such a servo amplifier becomes also quite easy. Considering the potential high performance, the servo amplifiers with sinus-commutation will get more and more applications in the servo drives with maxon EC motors.

For the convenience to use the sinus servo amplifiers in the servo drive with a maxon motor, this document will explain how to calculate the important parameters of a maxon EC motor for the operation in sinus-commutation according to the parameters provided in the data-sheets.

## 2. Maximum continuous current

The maximum continuous current of a maxon-EC-motor is limited by the power losses in the motor. The power losses consist of the copper losses in the three-phase windings and iron losses (mainly eddy current losses proportional to the quadrate product of the flux-density and rotor speed). If only the copper losses are considered, the maximum continuous current of a maxon-EC-motor in sinus-commutation can be calculated according to the given value in data-sheet under item Max. continuous current measured in block-commutation. The maximum continuous current in sinus-commutation in this document is defined as the amplitude of the sinusoidal current in one of the three-phase windings.

The copper loss in the phase-windings in block commutation is,

$$P_{copper} = 2 \cdot R \cdot I_b^2 \qquad (1)$$

where R is the phase ohmic resistance, $I_b$ is the phase current in block-commutation. It is assumed that the phase ohmic resistance are the same in the three-phase windings.

The copper loss in the three-phase windings in sinus-commutation is,

$$P_{copper} = \frac{3}{2} \cdot R \cdot I_s^2 \qquad (2)$$

where, $I_s$ is the amplitude of the sinusoidal currents in a phase-winding.

From the formulas (1) and (2), the relation between the amplitudes of phase currents in block-commutation and sinus-commutation is,

$$I_s = \frac{2}{\sqrt{3}} \cdot I_b \qquad (3)$$

From formula (3), the maximum continuous current of a maxon-EC-motor can be estimated from the parametern provided in the data-sheet by considering only the copper power losses in the three-phase windings.

$$I_{s\max} = \frac{2}{\sqrt{3}} \cdot I_{b\max} \tag{4}$$

where, $I_{smax}$ and $I_{bmax}$ are the maximum continuous currents in sinus-commutation and block-commutation, respectively.

It must be noted that the amplitude of the sinusoidal current $I_s$ is used for the calculation instead of the effective value because the amplitude of the output current is more significant to the servo amplifiers and is also easy to be measured with an oscillator scope.

**3. Torque constant**

In the data-sheets of maxon-EC-motors, the torque constant is defined as the average ratio of the output magnetic torque to the constant dc current in two of three-phase windings over a block-commutation (60° electrical angle). Figure 1 shows the waveform of output magnetic torque of an EC motor controlled by a servo amplifier with block commutation.

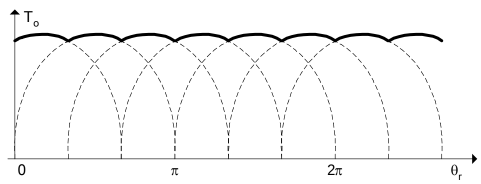


*Fig. 1 Waveform of magnetic torque of an EC motor in block-commutation*

When two of three-phase windings of an EC motor are fed with a DC current and the rotor rotates one revolution, the generated magnetic torque is a sinusoidal curve. At a rotor position, there are six possible combinations of two current-conducting phase windings among three-phase windings. In the operation of the block-commutation, the two current-conducting phase windings at a rotor position are selected so that the maximum torque for a given DC current can be generated. As the rotor rotates, the current will commutate between the phase windings with the help of Hall-sensors and a power converter. In a revolution, the output magnetic torque consists of segments of the six sinusoidal curves as shown with thick line in *Fig. 1*. The electrical angle of a segment is 60°.

The magnetic torque is proportional to the amplitude of DC current in phase windings, and the sinusoidal waveform of magnetic torque in a segment can be represented as,

$$T_o = K_b \times I_b \times \sin\theta_r \tag{5}$$

where, $I_b$ is the amplitude of the DC current, $T_o$ is the generated magnetic torque, $\theta_r$ is the rotor position, $K_b$ is the torque constant that is the ratio of the peak value of the output torque in the segment to the amplitude of the dc current.

The average torque is calculated as,

$$\overline{T}_o = \frac{3}{\pi} K_b \times I_b \tag{6}$$

or

$$\overline{T}_o = \overline{K}_b \times I_b \tag{7}$$

where $\overline{K}_b = \frac{3}{\pi} K_b$

In formula (7), the average torque constant $\overline{K}_b$ is one in the data-sheets of maxon EC motors.

Different from considering only the DC current in two of three-phase windings to calculate the magnetic torque in block-commutation, the currents in three-phase windings must be considered in an EC motor fed by a servo amplifier with sinus-commutation. Besides, the peak amplitude of the sinusoidal waveforms of currents in three-phase windings will be used to calculate the magnetic torque instead of the DC current in block-commutation.

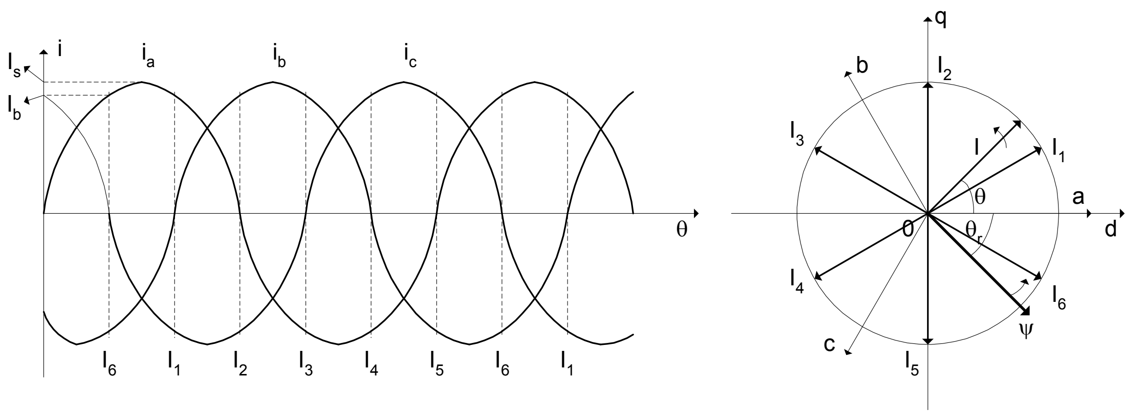


*Fig. 2 Current space vectors of a three-phase EC motor in a stator d-q reference frame*

The three-phase currents in the sinus-commutation are represented as,

$$\begin{aligned} i_a &= I_s \times \cos\theta \\ i_b &= I_s \times \cos(\theta - \frac{2\pi}{3}) \\ i_c &= I_s \times \cos(\theta + \frac{2\pi}{3}) \end{aligned} \tag{8}$$

For the calculation of the output magnetic torque in a three-phase ac motor, it is generally to use the 3/2 transformation to get the current vector in the stator d-q reference frame from the three phase currents as shown in *Fig. 2* right,

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} \tag{9}$$

From Eq. (8) and Eq. (9), the current vector is presented in the d-q reference as,

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \sqrt{\frac{3}{2}} \begin{pmatrix} I_s \cos(\theta) \\ I_s \sin(\theta) \end{pmatrix} \tag{10}$$

Since the current phase angle θ can be controlled continuously in the sinus commutation, the current vector will rotate in the reference frame continuously. In the air-gap, the magnetic flux is mainly dependent on the permanent magnet. The flux-vector ψ shown in Fig. 2 right rotates with the rotor in the reference frame. The vector can be represented as,

$$\begin{pmatrix} \psi_d \\ \psi_q \end{pmatrix} = \begin{pmatrix} \Psi \cos(\theta_r) \\ \Psi \sin(\theta_r) \end{pmatrix} \tag{11}$$

The magnetic torque will be got from Eq. (10) and Eq. (11) as,

$$T_o = k\left(i_q \psi_d - i_d \psi_q\right) = \sqrt{\frac{3}{2}} k\Psi I_s \sin(\theta - \theta_r) \tag{12}$$

where, k is a constant associated with the motor construction.

When the phase of the current is controlled so that it is always vertical to the flux vector, the magnetic torque in Eq. (12) will be,

$$T_o = \sqrt{\frac{3}{2}} k\Psi I_s = K_s I_s \tag{13}$$

where, $K_s$ is referred to as the torque constant in the sinus-commutation.

To compare the magnetic torque in the block commutation and sinus commutation, we can calculate the magnetic torque in the block commutation in the same way as we do for the sinus commutation. Studying the *Fig. 2*, the operating states in the block commutation are only the special cases of the sinus commutation. In the steady operation of the block commutation, six difference states of the current conduction in the three phase windings are corresponding to the states in the sinus current waveforms shown in *Fig. 2* left, where one phase current is zero. The current vectors in these states are shown in the d-q reference frame as $I_1$ to $I_6$. In the block commutation, the current vector in the d-q reference frame jumps from a state to another, instead of rotating continuously with the rotor position. It means that the current vector in the block commutation is not always vertical to the flux vector, and hence, the output torque will change with the rotor position even thought the current amplitude is kept constant. The average magnetic torque in the block commutation can be calculated with Eq. (12) by considering that the current vector will stay at a position as the rotor rotates 60° electrical angle,

$$\overline{T}_o = \frac{3}{\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{\frac{3}{2}} k\Psi I_s \sin(\theta - \theta_r) d(\theta - \theta_r) = \frac{3}{\pi} \sqrt{\frac{3}{2}} k\Psi I_s \tag{14}$$

Instead of the peak value of the sinus current, the phase current at the states $I_1$ to $I_6$ in the block commutation is utilized and Eq. (14) will become to,

$$\overline{T}_o = \frac{3}{\pi} \sqrt{2} k\Psi I_b = \overline{K}_b I_b \tag{15}$$

It is equal to the magnetic torque given in Eq. (7). From Eq. (13) and (15), the relation of the torque constant is obtained as,

$$\frac{K_s}{\overline{K}_b} = \frac{\pi}{2\sqrt{3}} \approx 0.9$$

Or, the output magnetic torque in the sinus commutation can be calculated with the torque constant measured in the block commutation with the following formula,

$$T_o = \frac{\pi}{2\sqrt{3}} \overline{K}_b I_s \approx 0.9 \overline{K}_b I_s \tag{16}$$

**4. Maximum output voltage**

The control of the three-phase voltage source inverter (VSI) is based on the space vector ( SV) PWM-scheme. The three-phase VSI shown in *Fig. 3* left can be analyzed with the help of a two phases equivalent system with the transformation as,

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} \tag{17}$$

All the possible switching states of a three-phase VSI can be represented with the six non-zero voltage vectors shown in *Fig. 3*, right. There are two states with zero output phase-to-phase voltage, all the high-side power switches are switched for one zero output voltage vector, and all the low-side power switches are switched on in another zero output voltage vector. In the SV PWM, the eight basic vectors are used to modulate the output voltages to get the continuously changing voltage vector. When the output voltage vector in the reference frame d-q shows a circle, the output phase-to-phase voltage will be in the sinusoidal waveform according to the transformation given in Eq. (17). Therefore, the maximum amplitude of the output sinusoidal phase-to-phase voltage will be corresponding to the maximum circle in the hexagon, and it is equal to dc-link voltage $U_{dc}$ in the case that the maximum ratio is 1.

By considering the modulation ratio σ, the amplitude of the output sinusoidal phase-to-phase voltage is,

$$\left|U_{pp\sigma}\right| = \sigma U_{dc} \tag{18}$$

In practical application, the modulation ratio is always limited by a minimum pulse-width and hence, can not reach 100%. The minimum time for the state of zero-voltage space vector output is required in which the charge pump of the gate driver of power MOSFETs and optical current detectors work properly and the AD converter will convert the analog signal to digital value without serious disturbance. In the sinus-servo amplifier DES 50/5, the minimum time is 2 µs. The maximum modulation ratio $\sigma_{max}$ at the PWM switching frequency of 50 KHz is 90%.
the maximum amplitude of voltage space vector is calculated as,

$$\left|U_{pp\,max}\right| = 0.9 \cdot U_{dc} \tag{19}$$

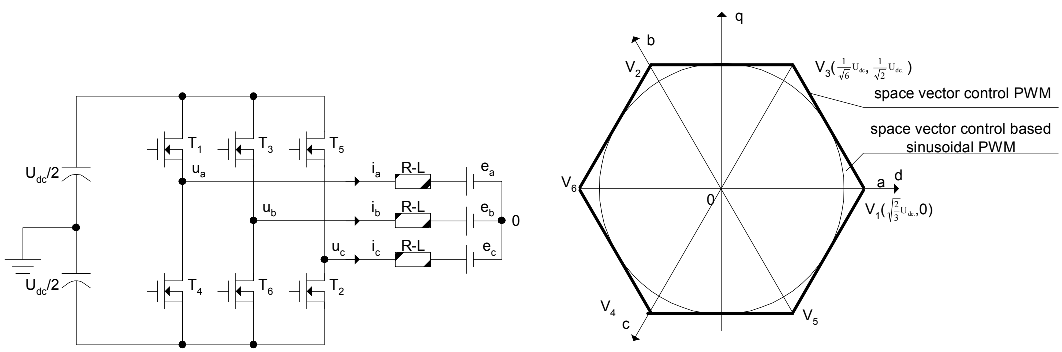


*Fig. 3 Representation of voltage space vector in a stator α-β reference frame*

## 5. Speed constant

Speed constant indicates the theoretical speed per volt without considering the friction and load torque. It can be used to estimate the no-load speed corresponding to the input voltage of a maxon-EC-motor. Like the torque constant, the speed constant of a maxon-EC-motor is defined for the application in the block-commutation. The active waveform of the phase-to-phase electromotive force is the same as the torque waveform shown in *Fig. 1*, and the average value the electromotive force is used to define the speed constant. However, in the sinus-commutation, the peak value of the electromotive force must be considered. The output voltage of a servo amplifier in sinus-commutation must overcome the peak value of the electromotive force to drive the phase currents in the sinusoidal waveforms. For this reason, the estimation of the theoretical speed without any load torque must consider the peak value of the electromotive force as,

$$n_r = k_n \cdot \left|U_{pp\sigma}\right| = \frac{3}{\pi}\bar{k}_n \cdot \left|U_{pp\sigma}\right| \tag{20}$$

where, $k_n$ is the speed constant in sinus-commutation, and $\bar{k}_n$ is the speed constant in block-commutation given in the data-sheets of maxon-EC-motors.

## 6. Output power of a servo amplifier in sinus-commutation

When the phase currents are sinusoidal waveforms, the output power of a servo amplifier with sinus-commutation is calculated according to Eq. (18) as,

$$P_o = \frac{\sqrt{3}\cdot\sigma\cdot U_{dc}\cdot I_s}{2} \qquad (21)$$

The maximum output power will be,

$$P_o = 0.78\cdot U_{dc} I_{s\,max} \qquad (22)$$

where, $I_{smax}$ is the maximum peak value of output phase currents.

## 7. Conclusion

The parameters of maxon-EC-motors in the data-sheets are measured for the applications with servo amplifiers in the block-commutation. For the application of servo amplifiers with sinus-commutation, the parameters must be treated properly, either to convert the parameters for the sinus-commutation case, or use the special formulas for the calculation.

The important formulas are listed in the following,

The relation between the maximum continuous currents in sinus-commutation and block-commutation is,

$$\boxed{I_{s\,max} = \frac{2}{\sqrt{3}}\cdot I_{b\,max}}$$

The relation between the torque constants in sinus-commutation and block-commutation is,

$$\boxed{K_s = \frac{\pi}{2\sqrt{3}}\overline{K}_b}$$

The formula to calculate the magnetic torque in sinus-commutation with the torque constant from the data-sheets of maxon-EC-motors is,

$$\boxed{T_o \approx 0.9\overline{K}_b I_s}$$

The maximum output voltage of a three-phase inverter in sinus-commutation is,

$$\boxed{\left|U_{pp\,max}\right| \approx 0.9\cdot U_{dc}}$$

The relation between the speed constants in sinus-commutation and block-commutation is,

$$\boxed{k_n = \frac{3}{\pi}\overline{k}_n}$$

The maximum output power of a three-phase inverter in sinus-commutation is,

$$\boxed{P_o = 0.78\cdot U_{dc} I_{s\,max}}$$